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THEORETICAL NOTES ON BUBBLES AND THE CURRENT CRISIS

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We dedicate this research to the memory of Paul Samuelson, the best economist of the twentieth century, and the first one to understand that pyramid schemes are possible and might raise welfare even if we are all rational and well informed. We thank Fernando Broner and Francesco Caselli for comments on an earlier draft, and Stavros Panageas for a very helpful discussion. We also thank Robert Zymek for excellent research assistance. We acknowledge financial support from the European Research Council, the Lamfalussy Program sponsored by the European Central Bank, the Spanish Ministry of Science and Innovation, the Generalitat de Catalunya, and the Barcelona GSE Research Network. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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Theoretical Notes on Bubbles and the Current Crisis
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ABSTRACT

We explore a view of the crisis as a shock to investor sentiment that led to the collapse of a bubble or pyramid scheme in financial markets. We embed this view in a standard model of the financial accelerator and explore its empirical and policy implications. In particular, we show how the model can account for: (i) a gradual and protracted expansionary phase followed by a sudden and sharp recession; (ii) the connection (or lack of connection!) between financial and real economic activity and; (iii) a fast and strong transmission of shocks across sectors and countries. We also use the model to explore the role of fiscal policy

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History shows that capitalist economies alternate expansions and recessions. Thus, even in the heights of the expansion that went from the mid 1990s to the subprime mortgage crisis in the summer of 2007 it was widely understood that a crisis would someday hit the world economy. But nobody anticipated what has happened since. The depth of the current recession and the blazing speed with which it has propagated across industries and countries far exceeds even the most pessimistic scenarios. In fact, we need to go back to the Great Depression of the 1930s to find a crisis of a similar magnitude and global scope. It is still not clear however that the lessons we learned from that earlier crisis are useful to understand what is going on today.

As everybody else, macroeconomists have been taken by surprise by the unfolding of events. Even worse, providing an accurate diagnosis of the problem and coming up with clear-cut policy prescriptions is turning out to be a really hard challenge. Part of the reason for this, of course, is that the state-of-the-art macroeconomic models used for policy analysis are poorly adapted to this task. These models typically emphasize nominal rigidities and labor market frictions, and downplay the role of financial frictions. As a profession, we must go back to the drawing board and reverse these priorities. To understand the current crisis we need models that bring back financial frictions to center stage.

Recent attempts to do this build on the seminal contributions by Bernanke and Gertler (1989) and Kiyotaki and Moore (2007) who developed models of the "financial accelerator" mechanism.[1] These models were designed to show how financial frictions amplify the impact of traditional macroeconomic shocks through their effects on net worth. The intuition is simple: the role of financial markets is to intermediate funds from those that have them (i.e. the savers or creditors) to those who know what to do with them (i.e. the entrepreneurs or borrowers). This intermediation is useful because it raises the average efficiency of the economy and thus the welfare of its inhabitants. To be able to do this intermediation, savers need guarantees from entrepreneurs that the funds they lend them (plus an attractive enough return!) will be paid back once the investments give their fruits. The net worth of entrepreneurs, i.e. the amount of future funds that they can pledge today to creditors, is akin to those guarantees. When net worth is low, entrepreneurs cannot borrow enough and the economy operates at low levels of efficiency. When net worth is high, entrepreneurs can borrow enough and the economy operates at high levels of efficiency.

[1] Of course, these initial models were quite stylized. Carlstrom and Fuerst (1997) and Bernanke et al. (1999) developed more sophisticated versions for quantitative analysis. Recently, Gertler and Kiyotaki (2010) and Fernandez-Villaverde and Ohanian (2010) have used versions of this model to study the current crisis.

There are two alternative ways of using the financial accelerator model to think about the current crisis. The first one is based on the notion that, as a result of unprecedented changes in the financial system, the financial accelerator mechanism has become very powerful at amplifying traditional macroeconomic shocks. Consequently, these shocks can now unleash massive contractions of credit and deep recessions. The problem with this view, however, is that it seems difficult to identify the specific shock that has thrown the world economy into such a severe recession.

A second way of using the model is based on the notion that, instead of a traditional macroeconomic shock, the world economy has suffered a large financial shock that has drastically reduced net worth. To articulate this view we need to develop a rigorous model of such shocks. This is our main goal in writing these notes. We show how, in the financial accelerator model, changes in investor sentiment affect the market valuation of firms and therefore their net worth. When investors are optimistic, firm prices contain bubbles. These bubbles are useful because they raise net worth, leading to a credit expansion and a boom. When investors become pessimistic, these bubbles burst and net worth falls, leading to a credit contraction and a recession.

This shift in perspective is more than academic exercise. On the empirical side, introducing bubbles in the model allows us to provide simple unified narrative of the main macroeconomic developments of the recent past and the current crisis as a bubbly episode that started in the early 1990s and ended in 2007-08. Moreover, introducing bubbles also provides answers to two burning questions for current macroeconomics: (i) Why do asset (stock, housing, ...) prices fluctuate so much and in ways that seem so unrelated to fundamentals? and (ii) How is it that the current crisis has propagated so quickly and so strongly across sectors and countries?

On the policy side, modelling the crisis as the collapse of a bubble affects the role of policy as a stabilization tool. The case for a fiscal stimulus package and its optimal design depend crucially on whether the shock that led to the crisis is a traditional macroeconomic shock or a shock to investor sentiment. If the latter, we describe the type fiscal package that can get the world economy out of the crisis. Whether this package is feasible, though, depends on the credibility of the government.

In thinking about the origin and consequences of the current crisis, there are different, but complementary, lines of research that can be pursued. One approach is to focus on the particular details and institutional arrangements of financial markets, emphasizing the role of specific features – like regulation or the incentives of certain market participants – in generating and fueling the crisis.[2] An alternative approach is to take a step back and think instead of the general features that

[2] For such an account, see Brunnermeier (2009).

have characterized financial markets, and generally macroeconomic performance, in recent years. This approach, which we adopt in these notes, is also followed in recent papers by Gertler and Kiyotaki (2009) and Caballero, Farhi and Gourinchas (2008). As mentioned already, Gertler and Kiyotaki draw on the insights of the financial accelerator literature in order to interpret the current crisis. Caballero, Farhi and Gourinchas instead relate the crisis to the "global imbalances" of recent years and, in particular, to the prominent role of the United States as a provider of financial assets for the world economy. They argue that large capital flows towards the United States led to the creation of asset bubbles that eventually burst, giving rise to the subprime crisis.

Methodologically, we build on the traditional literature on rational bubbles that goes back to Samuelson (1958). Tirole (1985) analyzed the conditions for the existence of such bubbles in the context of a production economy. Our model is very close to Tirole's with the difference that, in our setup, the presence of financial frictions implies that bubbles can be expansionary and increase credit and output. This finding is related to recent results by Caballero and Krishnamurthy (2006), Kraay and Ventura (2007), and Farhi and Tirole (2009). Our framework differs from these last papers in two crucial respects, though. The first is that we study expansionary bubbles in the context of a standard production economy. The second is that, as in Martin and Ventura (2010), bubbles in our setting can arise even if all investments are dynamically efficient in the economy's fundamental equilibrium.

These notes are organized as follows. Section 1 develops a stylized version of the financial accelerator model and explores the effects of traditional macroeconomic shocks. Section 2 shows that the model has additional equilibria with bubbly episodes and uses them to interpret the crisis. Sections 3 and 4 extend the framework to study how bubbly episodes can lead to contagion, and how policy can react to the bursting of a bubble. Section 5 concludes.

1 A canonical model of financial frictions and business cycles

In a recent paper, Gertler and Kiyotaki (2009) develop a "canonical framework to help organize thinking about credit market frictions and aggregate economic activity in the context of the current crisis" (p.1). This framework is built around an agency cost that limits the ability of firms to pledge future resources to their creditors. This section develops a stripped-down version of this framework and uses it in the way that Gertler and Kiyotaki suggest.

1.1 Basic setup

Our model builds on Samuelson's two-period overlapping-generations structure. The economy contains an infinite sequence of generations, indexed by $t \in (-\infty, +\infty)$. Each generation contains a continuum of individuals of size one, indexed by $i \in I_t$. Individuals maximize expected old-age consumption, i.e. $U_{it} = E_t \{c_{it+1}\}$; where U_{it} and c_{it+1} are the utility function when young and the old-age consumption of individual i of generation t. To finance their consumption, individuals supply one unit of labor when young. Since individuals only care about old age consumption, they save their entire labor income. Since individuals are risk-neutral, they always invest their savings so as to maximize their expected return.

The economy also contains an infinite sequence of generations of firms, indexed by $j \in J_t$. The set J_t contains all firms that were created, in period t or before and are still operating. Firms produce output with a Cobb-Douglas technology: $F(l_{jt}, k_{jt}) = l_{jt}^{1-\alpha} \cdot k_{jt}^{\alpha}$; where l_{jt} and k_{jt} are the labor and capital used by firm j in period t. Firms also produce capital with a technology that uses one unit of output in period t to produce A_{jt} units of capital in period $t+1$. The capital stock of firm j evolves as follows:

$$k_{jt+1} = A_{jt} \cdot I_{jt} + (1-\delta) \cdot k_{jt}, \tag{1}$$

where I_{jt} is the investment of firm j, and $\delta \in [0,1]$ is the rate of depreciation. To motivate the need for intermediation, we make two assumptions about the life cycle of firms. The first one is that investment efficiency is high when a firm starts and then stabilizes at a lower level when it becomes mature:

$$A_{jt} = \begin{cases} \pi_t & \text{if } j \in J_t^N \\ 1 & \text{if } j \notin J_t^N \end{cases}, \tag{2}$$

where J_t^N is the set of "new" firms in period t, i.e. the set of firms that are created in period t and start producing output in period $t+1$. We refer to π_t as the investment efficiency and assume that it fluctuates randomly with $\pi_t > 1$. The second assumption is that only a subset I_t^E of generation t is capable of starting a firm. We refer to this subset as the "entrepreneurs" and assume that it has measure $\varepsilon \in [0,1]$. Everybody can manage an old firm.

Workers and savings are allocated to firms in the labor and financial markets. The labor market is competitive and all workers and firms can trade in it with zero or negligible transaction costs. Maximization then implies that:

$$l_{jt} = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} \cdot k_{jt}, \tag{3}$$

where w_t is the wage rate per unit of labor. Since the aggregate supply of labor is one, market clearing implies that:

$$w_t = (1 - \alpha) \cdot k_t^{\alpha}, \tag{4}$$

where $k_t \equiv \int_{j \in J_t} k_{jt}$ is the aggregate capital stock. Equation (3) is the labor demand of firm j, which results from hiring labor until its marginal product equals the wage. Since all firms use the same capital-labor ratio, this must be the aggregate one. Thus, Equation (4) says that the wage equals the marginal product of labor evaluated at the aggregate capital-labor ratio.

We turn next to the key piece of the model, namely, the financial market. This market consists of a credit market where individuals lend to firms, and a stock market where individuals buy and sell old firms. Both markets are competitive and all savers and firms can trade in them with zero or negligible transaction costs. We introduce however an agency cost that limits the ability of firms to obtain credit. In particular, we assume that firms can commit or pledge to their creditors only a fraction ϕ_t of their resources in period t. We refer to ϕ_t as the financial friction and assume that it fluctuates randomly within the unit interval. We adopt the convention that, in period t, individuals know the realization of shocks with index t (i.e. π_t and ϕ_t), but they do not know the realizations of shocks with index $t+1$ (i.e. π_{t+1} and ϕ_{t+1}). The resources of the firm in period $t+1$ consist of the revenue from sales net of labor costs, i.e. $F(l_{jt+1}, k_{jt+1}) - w_{t+1} \cdot l_{jt+1}$, plus the firm's resale or market value, i.e. V_{jt+1}. Therefore, we have that in each possible state of nature in period $t+1$ the following constraint holds:

$$R_{t+1} \cdot f_{jt} \leq \phi_{t+1} \cdot [F(l_{jt+1}, k_{jt+1}) - w_{t+1} \cdot l_{jt+1} + V_{jt+1}], \tag{5}$$

where f_{jt} is the credit that firm j obtains in the credit market in period t, and R_{t+1} is the (gross) ex-post return to loans. We allow R_{t+1} to be contingent on any variable which is known in period $t+1$, and refer to $E_t R_{t+1}$, as the interest rate. The right-hand side of Equation (5) captures the concept of net worth. That is, the amount of future resources that firms can use as a collateral to obtain credit today. The shock ϕ_t captures the quality of the legal system and other institutional arrangements that support credit.

Maximization implies that non-entrepreneurs will lend and buy old firms simultaneously if and only if the expected return to owning an old firm equals the interest rate:[3]

$$E_t R_{t+1} = \max_{\langle I_{jt}, f_{jt}\rangle} \frac{E_t\left\{\alpha \cdot k_{t+1}^{\alpha-1} \cdot \left[A_{jt} \cdot I_{jt} + (1-\delta) \cdot k_{jt}\right] - R_{t+1} \cdot f_{jt} + V_{jt+1}\right\}}{V_{jt} + I_{jt} - f_{jt}} \quad \text{if } j \notin J_t^N. \tag{6}$$

To compute the return to owning an old firm, note that in period t the owner must spend the purchase price plus the cost of new capital minus credit. Then, in period $t+1$ the owner obtains the revenue from sales net of labor and financing costs plus the resale value of the firm. Maximization also implies that entrepreneurs start new firms only if the expected return to doing so is not lower than the interest rate:

$$E_t R_{t+1} \leq \max_{\langle I_{jt}, f_{jt}\rangle} \frac{E_t\left\{\alpha \cdot k_{t+1}^{\alpha-1} \cdot A_{jt} \cdot I_{jt} - R_{t+1} \cdot f_{jt} + V_{jt+1}\right\}}{I_{jt} - f_{jt}} \quad \text{if } j \in J_t^N. \tag{7}$$

Unlike old firms, new firms start without capital and their owners, who are also their creators, do not have to pay a price for them, i.e. $k_{jt} = V_{jt} = 0$ if $j \in J_t^N$.

The next step is to determine the interest rate and firm prices that clear the credit and stock markets. We conjecture that

$$E_t R_{t+1} = \alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta, \tag{8}$$

$$V_{jt} = (1-\delta) \cdot k_{jt}, \tag{9}$$

and then verify that this conjecture is indeed consistent with market clearing. Equation (8) says that the interest rate equals the return to producing a unit of capital within an old firm. Equation (9) says that the price of a firm equals the cost of replacing the capital that it owns. Ideally, all investment should take place within new firms, as these have a technological advantage when producing new capital. This is not possible however if the financial friction is severe enough. The conjecture in Equations (8) and (9) turns out to be correct if the equilibrium is inefficient and some investment is carried out within old firms.

At the proposed interest rate and firm prices, entrepreneurs strictly prefer to start new firms than to lend or purchase old firms. Moreover, since the interest rate is below the return to investing in new firms the owners of these firms ask for as much credit as possible. The optimal financing contract ensures that Equation (5) is binding in all states of nature. Adding these constraints

[3] Here, we have used that Equations (3) and (4) imply that $F(l_{jt}, k_{jt}) - w_t \cdot l_{jt} = \alpha \cdot k_t^{\alpha-1} \cdot k_{jt}$.

across states of nature, we find that:[4]

$$f_{jt} = \frac{1}{1 - E_t\phi_{t+1} \cdot \pi_t} \cdot E_t \left\{ \phi_{t+1} \cdot \pi_t \cdot w_t \right\}. \tag{10}$$

Not surprisingly, credit increases with the wealth of entrepreneurs and their investment efficiency, and decreases with the financial friction.

At the proposed interest rate and firm prices, non-entrepreneurs are indifferent between lending and purchasing old firms. If they choose the latter, they are also indifferent regarding the amount of investment and external financing of their firms. As a group, non-entrepreneurs purchase the stock of old firms, give credit to new firms and use any savings left to produce new capital within their old firms. To verify that markets clear, we must check that this group has enough savings to do all of this:

$$(1 - \varepsilon) \cdot w_t - f_t^N \geq V_t, \tag{11}$$

where $V_t \equiv \int_{j \notin J_t^N} V_{jt}$ and $f_t^N \equiv \int_{j \in J_t^N} f_{jt}$. We assume from now on that this condition holds and, as a result, the conjectured interest rate and firm prices are verified.[5]

Aggregating Equation (1) across firms, we find that:[6]

$$k_{t+1} = \left[1 + \frac{(\pi_t - 1) \cdot \varepsilon}{1 - E_t\phi_{t+1} \cdot \pi_t} \right] \cdot (1 - \alpha) \cdot k_t^{\alpha}. \tag{12}$$

Equation (12) is the law of motion of the capital stock. The dynamics of this economy are akin to those of a Solow model with shocks to the average efficiency of investment. From any initial capital stock, the economy converges towards a steady state in which the capital stock fluctuates

[4] Adding up Equation (5) across states of nature yields:

$$E_t R_{t+1} \cdot f_{jt} = E_t \left\{ \phi_{t+1} \cdot \left[\alpha \cdot k_{t+1}^{\alpha-1} \cdot \pi_t \cdot (w_t + f_{jt}) + V_{jt+1} \right] \right\},$$

where we have used that: (i) Equations (3) and (4) imply that $F(l_{jt}, k_{jt}) - w_t \cdot l_{jt} = \alpha \cdot k_t^{\alpha-1} \cdot k_{jt}$; and (ii) entrepreneurs put all of their savings in the firm and Equations (1) and (2) then imply that $k_{jt+1} = \pi_t \cdot (w_t + f_{jt})$. To obtain Equation (10), we substitute in the conjectured interest rate and firm prices and solve for f_{jt}.

[5] This requires that:

$$\frac{1 - E_t\phi_{t+1} \cdot \pi_t - \varepsilon}{1 - E_t\phi_{t+1} \cdot \pi_t} \cdot (1 - \alpha) \cdot k_t^{\alpha} \geq (1 - \delta) \cdot k_t.$$

In terms of the primitives of the model, this implies that: (i) $E_t\phi_{t+1} \cdot \pi_t < 1 - \varepsilon$ in all dates and states of nature, and (ii) δ is high enough. The first restriction ensures that the credit constraint is tight enough so that, after giving credit to new firms, non-entreprenurs still have some savings left in their hands. The second restriction ensures firm prices are sufficiently low so that these savings are sufficient to purchase the stock of old firms.

[6] Investment spending consists of the savings of the young minus their purchases of old firms, i.e. $w_t - V_t = (1 - \alpha) \cdot k_t^{\alpha} - (1 - \delta) \cdot k_t$. Of this total, new firms invest $\frac{\varepsilon}{1 - E_t\phi_{t+1} \cdot \pi_t} \cdot (1 - \alpha) \cdot k_t^{\alpha}$ with efficiency π_t, while the rest is invested by old firms with efficiency one.

within a range which is defined by the support of the shocks. These shocks might originate in the investment technology (π_t) or the financial friction (ϕ_t), but have similar macroeconomic effects as they both work through the average efficiency of investment.

1.2 Looking to the crisis through the lens of the canonical model

We are ready to use the canonical model in the way that Gertler and Kiyotaki suggest, namely, as a framework to help organize our thinking about the current crisis. The stylized facts are well known, of course (see Figure 1). The world economy entered a long and steady expansion around the mid 1990s, with increases in consumption and investment. The prices of stocks, real state and other assets grew to unprecedented levels. Intermediation soared, while interest rates fell to historical lows. This expansion lasted more than a decade, leading many to think that the business cycle was over. This might have been too optimistic. But nobody anticipated what happened after the summer of 2007: a sudden and sharp drop in stock and real state prices, a massive collapse in intermediation and the worse financial crisis since the Great Depression. Since then, investment has come to a halt and the world economy has experienced negative growth. We are only now starting to see the light at the end of the tunnel.

The key question, of course, is how did all this happen. Coming up with a convincing explanation for such a sharp and unexpected change in economic outcomes is a fascinating academic challenge with far reaching policy implications. At a deep level, explanations of the crisis fall into one of two rough categories. The first one includes explanations based on the notion that something fundamental or technological has happened. These explanations emphasize aggregate resource constraints and view the crisis as a negative shift of these constraints. A second set of explanations start from the premise that nothing fundamental has changed, and that we are only witnessing a massive coordination failure. This second set of explanations emphasize the role of expectations and view the crisis as a negative shift in those.

The canonical model described above offers two alternative, but complementary, explanations of the crisis: a shock to the investment technology, π_t; and a shock to the financial friction, ϕ_t. Both of these shocks are fundamental or technological, although they originate in different parts of the economy: the corporate or the financial sector, respectively. We consider each of them in turn.

Figure 2 shows the response of the economy to a transitory shock to the investment technology.[7]

[7] In particular, we assume that $\pi_t = \bar{\pi}$ if $0 \leq t < T$ and $\pi_t = \pi$ for all $t < 0$ and $t \geq T$, with $\bar{\pi} > \pi$. To allow for a clean experiment, we assume that $\phi_t = \phi$ for all t, and that the economy was already in the steady state in period

The different panels plot the assumed path for the shock (π_t) and the responses of the capital stock (k_t), consumption (c_{t+1}), the stock market (V_t), the interest rate ($E_t R_{t+1}$) and intermediation (f_t^N).[8] All variables are shown as deviations from the steady state. The increase in π_t raises the average efficiency of investment through two channels. For a given allocation of investment, new firms become more efficient at investing. In addition, their net worth increases, relaxing their credit constraint and allowing them to do a larger share of the economy's investment. The increase in the average efficiency of investment shifts the law of motion of the capital stock upwards and the economy starts transitioning towards a higher steady state. As this happens, the capital stock and consumption increase. In the financial market, the interest rate declines, while intermediation and firm prices increase. When π_t goes back to its original level, all these changes unwind. The law of motion of the capital stock goes back to its original shape and the capital stock starts declining. Eventually, the economy goes back to its original steady state.

Figure 3 shows the response of the economy to a transitory shock to the financial friction.[9] We have calibrated the shocks so that the quantitative effect on the average efficiency of investment is the same in Figures 2 and 3. The most remarkable aspect of Figure 3 is that it is almost a carbon copy of Figure 2. The only difference between these figures is that Figure 3 shows a larger increase in intermediation. The reason is that shocks to the financial friction only affect the average efficiency of investment through one channel: the net worth of firms increases, relaxing their credit constraint and improving the allocation of investment. This is why a shock to ϕ_t requires a larger increase in intermediation than a shock to π_t to obtain the same increase in the average efficiency of investment. Since shocks to π_t and ϕ_t are observationally equivalent from a macroeconomic perspective, the only way to tell them apart would be to use microeconomic data to find out whether aggregate fluctuations in the average efficiency of investment are due to firms being more productive or having better access to credit.

The model is stylized and much work remains to be done to get it ready for serious quantitative analysis. But Figures 2 and 3 already show that it is possible to write down a model based on fundamental or technological shocks to the corporate (i.e. π_t) and/or the financial sector (i.e. ϕ_t) that delivers dynamics that are qualitatively consistent with the evidence. Moreover, the notion

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$t = 0$.

[8] The response of output and wages mimics that of the capital stock.

[9] In particular, we assume that $E_t\phi_{t+1} = \bar{\phi}$ if $0 \leq t < T$ and $E_t\phi_{t+1} = \phi$ for all $t < 0$ and $t \geq T$, with $\bar{\phi} > \phi$. To allow for a clean experiment, we assume that $\pi_t = \pi$ for all t, and that the economy was already in the steady state in period $t = 0$.

that it is a drop in aggregate net worth that has caused a collapse in intermediation is certainly appealing as it conforms to the perceptions of many observers and market participants.

Despite these encouraging signs, we are skeptical that a research strategy based on technological or fundamental shocks will eventually succeed at explaining the current crisis. A dramatic downturn as the one suffered by the world economy can only be caused by an equally dramatic shock. But we cannot see what is the specific technological shock that could have caused such a large change in the investment opportunities faced by firms. We also find it difficult to see what is the specific change in the institutional and/or technological framework of financial markets that has so suddenly left them so impaired to do their job. Neither the resources available for intermediation, nor the technology used for it seems to have changed much. A successful explanation of the crisis should tell us why we are producing less with unchanging resources.

Even before the current crisis, the large and unpredictable fluctuations in the stock and housing markets of recent years hardly mirrored the evolution of technological or fundamental shocks.[10] A successful explanation of the macroeconomic events of the recent years should also tell us why financial and real activity are sometimes delinked, and consider the possibility that asset prices move in ways that are unrelated to fundamentals.

So we need an explanation of (i) why asset prices move in ways that are unrelated to fundamentals, and (ii) how these movements in asset prices can lead to fluctuations in production with unchanged resources. The key point of these notes is that this does not require changing the model, but only the way we look at it. We show this next.

2 Bubbles as pyramid schemes

What is the price of a firm? We showed that the canonical model has an equilibrium in which the price of a firm equals the cost that it would take to replace the capital it owns. This price is often referred to as the fundamental value of a firm, since it also equals the net present value of all the output that the capital owned by the firm will ever produce. But the canonical model has many other equilibria in which firm prices are above their fundamental value. It is customary to refer to these equilibria as bubbly, since they capture the notion of firms being overvalued or having a bubble. We use these equilibria to sketch an alternative explanation of the current crisis.

[10] Although the recent evolution of real state prices is perhaps too close to us to draw any definitive conclusions, the stock price boom and bust of the late 1990s, which has been widely studied, seems hard to attribute to movements in fundamentals. For a detailed discussion on this last point, see LeRoy (2004).

2.1 Setup with bubbles

We solve the model again, conjecturing that the interest rate is still given by Equation (8) but that firm prices are now given by:

$$V_{jt} = (1 - \delta) \cdot k_{jt} + b_{jt}, \tag{13}$$

where b_{jt} is the overvaluation or bubble in firm j. The assumption that firm prices equal their fundamental value can be expressed as the restriction that $b_{jt} = 0$ for all j and t. This restriction cannot be justified on a priori grounds but there is always an equilibrium in which it is satisfied, as we showed in the previous section. Equation (13) already points out to the first macroeconomic effect of bubbles: since firm prices are high, the amount of savings devoted to purchase the stock of old firms increases and this reduces the funds available for investment.

At the proposed interest rate and firm prices, entrepreneurs strictly prefer to start new firms than to lend or purchase old firms and, just as before, they ask for as much credit as possible:

$$f_{jt} = \frac{1}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot E_t \left\{ \phi_{t+1} \cdot \left(\pi_t \cdot w_t + \frac{b_{jt+1}}{\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta} \right) \right\}. \tag{14}$$

Equation (14) points out to the second macroeconomic effect of bubbles: since future firm prices are high, entrepreneurs are able to obtain more credit and this improves the allocation of investments.

Of course, not any stochastic process for b_{jt} can be part of an equilibrium. Broadly speaking, there are two restrictions or requirements that bubbles must satisfy. The first one is that bubbles should grow fast enough to be attractive. At the proposed interest rate and firm prices, non-entrepreneurs are indifferent between lending and purchasing old firms if and only if:

$$E_t R_{t+1} = \frac{E_t b_{jt+1}}{b_{jt}}. \tag{15}$$

Equation (15) says that the expected growth rate of bubbles must equal the interest rate. If the growth rate of the bubble were less than the interest rate, owning firms with a bubble would not be attractive. This cannot be an equilibrium. If the growth rate of the bubble exceeded the interest rate, non-entrepreneurs would want to borrow to purchase bubbly firms. This cannot be an equilibrium either. The requirement that all bubbles have the same expected growth rate does not mean that all bubbles be correlated. We shall come back to this important point in section 3.

The second requirement for a bubble to be part of the equilibrium is that it should not grow too fast. Otherwise, the aggregate bubble would eventually be too large for the young to be able to purchase it and markets would not clear. Knowing this, standard backward-induction arguments would rule out the bubble today. To verify that markets clear, we must check that non-entrepreneurs have enough savings to lend to entrepreneurs and purchase the stock of old firms. That, is, we must check that Equation (11) holds. We keep assuming that this condition holds and, as a result, the conjectured interest rate and firm prices are verified.[11]

Aggregating Equation (1) across firms, we find that:[12]

$$k_{t+1} = \left[1 + \frac{(\pi_t - 1)\cdot\varepsilon}{1 - E_t\phi_{t+1}\cdot\pi_t}\right]\cdot(1-\alpha)\cdot k_t^{\alpha} + \frac{\pi_t - 1}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot\frac{E_t\left\{\phi_{t+1}\cdot b_{t+1}^N\right\}}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta} - b_t - b_t^N, \quad (16)$$

where $b_t \equiv \int_{j\notin J_{t-1}/J_{t-1}^N} b_{jt}$ and $b_t^N \equiv \int_{j\in J_{t-1}^N} b_{jt}$. A comparison of Equations (12) and (16) shows that, in principle, the effect of bubbles on capital accumulation is ambiguous. The last two terms of Equation (16) shows that purchasing the existing bubble reduces capital accumulation by diverting resources away from investment. Since only non-entrepreneurs purchase bubbly firms and their investment efficiency is one, the existing bubble crowds out capital one to one. The second term of Equation (16) shows that the expected bubble expands capital accumulation by relaxing credit constraints, increasing intermediation and the average efficiency of investment. To understand this term, note that the expected bubble raises the net worth of efficient investors by $\frac{E_t b_{t+1}^N}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta}$, which enables them to expand borrowing by a factor of $\frac{E_t\phi_{t+1}}{1 - E_t\phi_{t+1}\cdot\pi_t}$, and each unit borrowed entails an efficiency gain of $\pi_t - 1$.

To complete the description of the dynamics of the economy, we need to determine the evolution of the aggregate bubble. Aggregating Equation (15) across firms, we find that:

$$E_t b_{t+1} = \left(\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta\right)\cdot\left(b_t + b_t^N\right). \quad (17)$$

That is, the aggregate bubble grows faster than the interest rate because of the creation of new

[11] This requires now that:

$$\frac{1 - E_t\phi_{t+1}\cdot\pi_t - \varepsilon}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot(1-\alpha)\cdot k_t^{\alpha} - \frac{1}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot\frac{E_t\left\{\phi_{t+1}\cdot b_{t+1}^N\right\}}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta} \geq (1-\delta)\cdot k_t + b_t + b_t^N,$$

where $b_t \equiv \int_{j\notin J_{t-1}^N} b_{jt}$ and $b_t^N \equiv \int_{j\in J_{t-1}^N} b_{jt}^N$. The presence of bubbles makes the condition more stringent. Bubbles raise both intermediation and the value of old firms, leaving less savings to produce capital within old firms.

[12] Investment spending consists of the savings of the young minus their purchases of old firms, i.e. $w_t - V_t = (1-\alpha)\cdot k_t^{\alpha} - (1-\delta)\cdot k_t - b_t - b_t^N$. Of this total, new firms invest $\frac{1}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot\left(\varepsilon\cdot(1-\alpha)\cdot k_t^{\alpha} + \frac{E_t\left\{\phi_{t+1}\cdot b_{t+1}^N\right\}}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta}\right)$ with efficiency π_t, while the rest is invested by old firms with efficiency one.

firms and, with them, new bubbles too. Any sequence for k_t, b_t and b_t^N that satisfies Equations (16) and (17) is an equilibrium, provided that Equation (11) holds in all dates and states of nature. The dynamics of this economy depend on the dynamics of firm prices, and we turn to these next.

2.2 Bubbly episodes

Bubbly episodes can take place in the canonical model. Generically, the economy fluctuates between periods in which $b_t = b_t^N = 0$ and periods in which $b_t > 0$ and/or $b_t^N > 0$. We say that the economy is in the fundamental state if $b_t = b_t^N = 0$. We say instead that the economy is experiencing a bubbly episode if $b_t > 0$ and/or $b_t^N > 0$. A bubbly episode starts when the economy leaves the fundamental state and ends the first period in which the economy returns to the fundamental state. Let $z_t \in \{F, B\}$ be a sunspot variable that determines the state of the economy. We refer to z_t as investor sentiment. We define the transition probabilities as $p_t = \Pr(z_{t+1} = F \,|z_t = B)$ and $q_t = \Pr(z_{t+1} = B \,|z_t = F)$. These transition probabilities could be a function of any endogenous or exogenous variable of the model, and could fluctuate randomly over time.

In the fundamental state, firm prices equal their fundamental values. Each period, there is some probability that a bubble episode starts in the new generation of firms. When this happens, an aggregate bubble appears and starts to grow according to Equation (17). This growth in the bubble is due to two factors: (i) as the new firms become old, their bubble keeps growing at an expected rate that equals the interest rate; and (ii) new bubbles appear in the successive generations of new firms. Throughout the bubbly episode, there is some probability that the episode ends and the economy reverts to the fundamental state. When this happens, all bubbles burst and firm prices go back to their fundamental values.

It turns out that this simple model can give rise to a wide array of equilibrium dynamics with bubbly episodes of different sorts.[13] To simplify the discussion, we consider next the simple example in which the probability of an episode ending is constant, i.e. $p_t = p$; and the rate of bubbly creation is also constant, i.e. $b_t^N = b^N > 0$ when the episode starts and then $b_t^N = n \cdot b_t$ until the episode ends, with $n > 0$. We also assume that q_t is small, so that the fundamental state is similar to the equilibrium of section 1. We use this example just for illustrative purposes. We also consider later examples in which p_t varies during a bubbly episode.

To be able to graphically describe the dynamics of the bubble during an episode, we further

[13] See Martin and Ventura (2010) for a full analysis of the set of equilibria in a related model.

simplify by assuming that there are no other type of shocks, i.e. $\pi_t = \pi$ and $\phi_t = \phi$. Moreover, if the rate of depreciation is large, i.e. $\delta \approx 1$, we can make the model recursive through a simple transformation of variables. Define x_t as the bubble's share of wealth or savings, i.e. $x_t \equiv \dfrac{b_t}{(1-\alpha)\cdot k_t^{\alpha}}$. Then, during a bubbly episode, we can rewrite Equation (17) as follows:

$$x_{t+1} = \frac{\dfrac{\alpha}{1-\alpha}\cdot\dfrac{1+n}{1-p}\cdot x_t}{1+\dfrac{(\pi-1)\cdot\varepsilon}{1-\phi\cdot\pi}+\left(\dfrac{(\pi-1)\cdot\phi\cdot n}{1-\phi\cdot\pi}-1\right)\cdot(1+n)\cdot x_t}, \tag{18}$$

if $z_{t+1} = B$ and $x_{t+1} = 0$ if $z_{t+1} = F$. Naturally, the derivation of Equation (18) assumes that Equation (11) holds. This condition can now be rewritten as follows:

$$x_t \leq \frac{1-\phi\cdot\pi-\varepsilon}{1-\phi\cdot(\pi-n)}\cdot(1+n)^{-1} \equiv \overline{x}. \tag{19}$$

The key observation is that the capital stock does not appear in Equations (18) and (19). Any path for x_t that that satisfies Equations (18) and (19) in all dates and states of nature is an equilibrium of the economy. Since $x_t = 0$ does this, we trivially have that such a path always exists. Of course, the interesting question is whether more paths are possible and, if so, how do these paths look like. Knowing this, we can then use Equation (16) to determine the associated paths for the capital stock. This allows us to interpret bubbly episodes literally as shocks to the law of motion of the Solow model.

Equations (18) and (19) embody the two requirements for bubbly episodes to be part of an equilibrium, and that we mentioned earlier. The first one is that the bubble must be expected to grow fast enough. Otherwise, holding the bubble would not be attractive and nobody would purchase it. This requirement is embodied in Equation (18), which is nothing but a restatement of Equation (15). The second requirement is that the bubble cannot be expected to grow too fast. Otherwise, it would eventually exceed available funds and it could not be purchased. Knowing this, standard backward-induction arguments would rule out the bubble today. This requirement is embodied in Equation (19) which is nothing but a restatement of Equation (11). Equations (18) and (19) can be used to show that bubbly episodes can happen if α is sufficiently low.

Within this example there are two types of bubbly episodes. The first type is the conventional or contractionary bubbly episode emphasized by Tirole (1985). These episodes occur in economies where investments are dynamically inefficient, and they require that $\frac{(\pi-1)\cdot\phi\cdot n}{1-\phi\cdot\pi} < 1$.[14] This condition ensures that bubbles have a negative effect on capital accumulation, as their negative impact on investment spending is not compensated by the increase in the average efficiency of investment. Bubbles raise the interest rate and reduce the capital stock. Figure 4 shows one of these contractionary episodes. The solid line depicts Equation (18) and the dotted line depicts the 45 degree line. The initial bubble must be in the interval $x_s \in [0, x^*]$. After the initial bubble appears, it declines as a share of wealth throughout. Only if the initial bubble is maximal, i.e. $x_s = x^*$; this rate of decline becomes zero.

The second type of bubbly episode is the non-conventional or expansionary one analyzed by Martin and Ventura (2010). These episodes arise in economies with financial frictions, and exist even if investments are dynamically efficient. These episodes require that $\frac{(\pi-1)\cdot\phi\cdot n}{1-\phi\cdot\pi} > 1$.[15] This condition ensures that bubbles have a positive effect on capital accumulation, as their negative impact on investment spending is compensated by the increase in the average efficiency of investment. These bubbles reduce the interest rate and increase the capital stock. Figure 5 shows one of them. The initial bubble can be anywhere the interval $x_s \in [0, \overline{x}]$. But these episodes now look quite different from the conventional ones. In particular, episodes might start with a small bubble that gains momentum over time. These bubbles can become very large before suddenly bursting.

We are ready to use these dynamics for firm prices to re-interpret the current crisis.

2.3 Looking to the crisis through the lens of the canonical model, again

The canonical model therefore offers a third explanation of the crisis: a shock to investor sentiment. Since non-conventional or expansionary bubbles are the only ones that stand a chance to be empirically relevant in the present situation, we focus on them in what follows. We would like to emphasize that we are not changing the model of the economy, but only the way to use it. Rather

[14] Episodes of this type exist if Equation (18) is below the 45 degree line for some $x_t \leq \bar{x}$. This requires that:

$$\frac{\alpha}{1-\alpha} \leq 1 + \frac{(\pi-1)\cdot\varepsilon}{1-\phi\cdot\pi}.$$

[15] Episodes of this type exist if Equation (18) is below the 45 degree line for some $x_t \leq \bar{x}$. This requires that:

$$\frac{\alpha}{1-\alpha} \leq \max_{p,n}\left\{\frac{1-p}{1+n}\cdot\left[1+\frac{(\pi-1)\cdot\varepsilon}{1-\phi\cdot\pi}+\left(\frac{(\pi-1)\cdot\phi\cdot n}{1-\phi\cdot\pi}-1\right)\cdot\frac{1-\phi\cdot\pi-\varepsilon}{1-\phi\cdot(\pi-n)}\right]\right\}.$$

than looking for fundamental or technological explanations such as shocks to π_t and ϕ_t, we instead look for an explanation that relies on a coordination failure by focusing on a shock to z_t.

Figure 6 shows the response of the economy to a shock to investor sentiment.[16] Once again, the different panels plot the assumed path for the shock (b_t) and the responses of the capital stock (k_t), consumption (c_{t+1}), the stock market (V_t), the interest rate ($E_t R_{t+1}$) and intermediation (f_t^N). We have calibrated the shock so that its effects on the capital stock are roughly the same as those of the technological shocks in Figures 2 and 3. The behavior of the different macroeconomic variables is similar to those in these previous figures. The main difference is that financial variables tend to fluctuate much more in the case of a shock to z_t. One reason is that the shock has a direct effect on firm prices that is absent in the case of shocks to π_t and/or ϕ_t. In addition, high asset prices reduce investment spending and this requires even a larger increase in intermediation to generate the same increase in the capital stock.

According this view, a bubbly episode is nothing but a macroeconomic pyramid scheme. The start of a bubble generates a positive wealth shock which can literally be described as a transfer from the future. This is a central feature of a pyramid scheme where the initiator claims that, by making him/her a payment now, the other party earns the right to receive a payment from a third person later. By successfully creating and selling a bubble, entrepreneurs assign themselves and sell the "rights" to the savings of a generation living in the very far future or, to be more exact, living at infinity. This appropriation of rights is a pure windfall or wealth gain for the entrepreneurs.

This wealth shock generates an efficiency gain, as it helps overcome the negative effects of the financial friction. The bubble increases the net worth of entrepreneurs and allows new firms to obtain more credit and invest more. In a very real sense, the bubble is like the oil that greases financial markets. The rights to the future generated by the bubble provide the collateralizable net worth that financial markets need to work efficiently. The bubble thus results in an increased average efficiency of investment. This is why the effects of a shock to investor sentiment resemble those of shocks to π_t and/or ϕ_t.

As a research strategy, viewing the current crisis as the bursting of a bubble seems to overcome the shortcomings of alternatives that rely on technological shocks. In particular, it explains (i) why asset prices move in ways that are unrelated to fundamentals; and (ii) why these movements in

[16] In particular, we assume that $z_t = B$ if $0 \leq t < T$ and $z_t = F$ for all $t < 0$ and $t \geq T$. To allow for a clean experiment, we assume that $\pi_t = \pi$ and $\phi_t = \phi$ for all t, and that the economy was already in the steady state in period $t = 0$.

asset prices can lead to fluctuations in production with unchanged resources. An additional benefit of this view is that it allows us to better understand why shocks are propagated so quickly and so strongly across industries and countries. We turn to this issue next.

3 Bubbles and contagion

It is common for expectations to experience large swings at the sector level: investors might, for example, become optimistic regarding the evolution of the high-tech sector, or they might become pessimistic regarding the prospects of the housing market. The effects of these sector-level shocks often spread throughout the whole economy, affecting other sectors directly through goods and factor markets but also indirectly through the market valuation of their firms. The same is true at the international level, where country shocks often propagate across the global economy with surprising speed and intensity: the recent crisis constitutes a case in point. To think about these issues, we introduce a second sector to our framework.

3.1 Setup with two sectors

Assume now that consumption and investment are a composite good made with two intermediates from different sectors, indexed by $s \in \{D, H\}$, where D stands for "dot-com" and H stands for "housing". Let y_t be the total amount of this composite or output produced in the world economy. Then, $y_t = \eta \cdot y_{Dt}^{\frac{1}{2}} \cdot y_{Ht}^{\frac{1}{2}}$, where y_{st} is the output of sector s, and $\eta > 0$ is a constant. We use the composite good as the numeraire.

Both sectors have many firms, indexed by $j \in J_{st}$, where $s \in \{D, H\}$ and $J_{Dt} \cup J_{Ht} \equiv J_t$. Sectors differ only in their factor intensities: $F(l_{jt}, k_{jt}) = l_{jt}^{1-\alpha_s} \cdot k_{jt}^{\alpha_s}$. We assume that $\alpha_D > \alpha_H$, so that the dot-com sector is capital-intensive relative to the housing sector. Each sector contains half of the entrepreneurs. Everybody can manage an old firm in either sector.

Adding a second sector does not affect the labor market, as the wage is still given by Equation (4), provided that we define α as the average share of labor, i.e. $\alpha \equiv \dfrac{\alpha_D + \alpha_H}{2}$. Once labor has been allocated to both sectors, we can write sectorial productions as a function of the aggregate capital stock:

$$y_{st} = A_s \cdot k_t^{\alpha_s}, \text{ for } s \in \{D, H\}, \tag{20}$$

where A_s is a sector-specific constant.[17]

We conjecture that the interest rate and firm prices are still given by Equations (8) and (13). At these interest rate and firms prices, entrepreneurs in both sectors strictly prefer to start new firms than to lend or purchase old firms. Also, they borrow as much as possible since the interest rate lies below the return to investing in new firms. Hence, total borrowing by entrepreneurs in sector $s \in \{,D, H\}$ is given by

$$f_{st}^{N} = \frac{E_t \phi_{t+1}}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot \left(\frac{\varepsilon}{2} \cdot \pi_t \cdot w_t + \frac{E_t b_{st+1}^{N}}{\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta} \right), \tag{21}$$

where $f_{st}^{N} \equiv \int_{j \in J_{st}^{N}} f_{jt}$ and $b_{st}^{N} \equiv \int_{j \in J_{st-1}^{N}} b_{jt}$ for $s \in \{D, H\}$.

At the proposed interest rate and firm prices, non-entrepreneurs are indifferent between purchasing old firms in either sector, investing in them, and lending to entrepreneurs if Equation (15) holds. To verify the conjectured interest rate and prices, we keep assuming that Equation (11) holds.

We can now describe the dynamics of this economy. Aggregating Equation (1), the law of motion of the aggregate capital stock k_t is still given by Equation (16). Equation (17) describing the dynamics of the aggregate bubble still applies. But it is useful to disaggregate these dynamics at the sector level:

$$E_t b_{st+1} = \left(\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta\right) \cdot \left(b_{st} + b_{st}^{N}\right), \text{ for } s \in \{D, H\}, \tag{22}$$

where $b_{st} \equiv \int_{j \notin J_{st-1}/J_{st-1}^{N}} b_{jt}$ for $s \in \{D, H\}$. Any sequence for k_t, b_{st} and b_{st}^{N} for $s \in \{D, H\}$ that satisfies Equations (16) and (22) is an equilibrium, provided that Equation (11) holds in all dates and states of nature. We examine some of these equilibria next.

3.2 Bubbly episodes

We are now ready to explore the effect of bubbles on the allocation of resources across sectors. To do so, we focus on the type of bubbly episodes analyzed in Section 2.2, which entail a constant rate of bubble creation n_s and a probability of bursting equal to p_s, although we now assume that $p_s = p_s(x_{st})$ for $s \in \{D, H\}$, with $p_s'(\cdot) \leq 0$, so that larger bubbles are more stable than smaller

[17] The appendix contains a derivation of the wage and the sectorial productions as functions of the aggregate capital stock.

ones.[18] Once again, we simplify the analysis by assuming that $\pi_t = \pi$ and $\phi_t = \phi$ and $\delta \approx 1$.

Using Equations (16) and (22) letting x_{st} and x_{st}^N denote the bubble of sector s as a share of wages, i.e. $x_{st} \equiv \dfrac{b_{st}}{(1-\alpha)\cdot k_t^\alpha}$ and $x_{st}^N \equiv \dfrac{b_{st}}{(1-\alpha)\cdot k_t^\alpha}$, we find that:

$$x_{st+1} = \frac{\dfrac{\alpha}{1-\alpha}\cdot\dfrac{1-p_s(x_{st})}{1+n_s}\cdot x_{st}}{1+\dfrac{(\pi-1)\cdot\varepsilon}{1-\phi\cdot\pi}+\sum\limits_{s'\in\{D,H\}}\left(\dfrac{\phi\cdot(\pi-1)\cdot n_{s'}}{1-\phi\cdot\pi}-1\right)\cdot(1+n_{s'})\cdot x_{s'}}, \tag{23}$$

for $z_{s,t} = B$. Equation (23) generalizes Equation (18) and illustrates how bubbly episodes in different sectors interact. If $x_{s't}$ is expansionary, it shifts the x_{st+1} mapping downwards. Intuitively, expansionary bubbles lower the interest rate and thus the rate at which other bubbles must grow in equilibrium. If instead $x_{s't}$ is contractionary, it shifts the x_{st+1} mapping upwards. Intuitively, contractionary bubbles raise the interest rate and thus the rate at which other bubbles must grow in equilibrium.

Naturally, the derivation of Equation (23) assumes that Equation (11) holds. This condition can now be rewritten as follows:

$$\sum_{s\in\{D,H\}} x_{ts}\cdot(1+n_s) \le \frac{1-\phi\cdot\pi-\varepsilon}{1-\phi\cdot(\pi-n)}. \tag{24}$$

The goal of this extension is to study how bubbles in different sectors interact with each other. Before doing this, however, we briefly review the effects of a single bubble in this two-sector economy. Assume, for instance, that a bubbly episode starts in the dot-com sector.[19] This has the usual effects: (i) it expands the net worth of new dot-com firms, enabling them to obtain more credit and raising the average efficiency of investment; (ii) it raises the price of old dot-com firms and this diverts resources away from investment. The former dominates the latter if and only if

$$\frac{\phi\cdot(\pi-1)\cdot n_D}{1-\phi\cdot\pi} > 1. \tag{25}$$

[18] The only reason for introducing this modification is to permit bubbles with different expected growth rates.

[19] This requires x_{Dt} to evolve according to Equation (23) and to have a stationary value that satisfies Equation (24). Formally, this requires that

$$\frac{\alpha}{1-\alpha} \le \max_n\left\{\frac{1}{1+n}\cdot\left[1+\frac{(\pi-1)\cdot\varepsilon}{1-\phi\cdot\pi}+\frac{1-\phi\cdot\pi-\varepsilon}{1-\phi\cdot\pi}\cdot\left(\frac{\pi\cdot\phi\cdot n_D}{1-\phi\cdot(\pi-n_D)}-1\right)\right]\right\}.$$

In this case, the bubbly episode in the dot-com sector expands the aggregate capital stock and output. As Equation (20) shows, this raises the output of both the dot-com *and* the housing sector. Although the bubbly episode takes place at the sector level, the whole economy expands. And, regardless of the sector in which the bubbly episode actually takes place, the capital-intensive sector is the one to expand the most.

This example illustrates how, by raising the net worth of new firms in their sector, expansionary bubbles free resources that are used to expand output throughout the economy. But bubbly episodes can also affect other sectors more directly through stock market prices in a way that resembles the phenomenon commonly known as "contagion". We now turn to this possibility.

3.3 Contagion

Consider first the case in which bubbly episodes complement and reinforce one another, generating what we label "positive contagion". This type of contagion happens when the episodes in question are of different types. To see this, assume that there is an expansionary bubbly episode in the dot-com sector and a contractionary one in the housing sector for which

$$\frac{\phi \cdot (\pi - 1) \cdot n_H}{1 - \phi \cdot \pi} < 1. \tag{26}$$

These two bubbles clearly have opposite effects on the aggregate economy. But the interesting thing is that they reinforce one another. From Equation (23), we can derive the stationary bubble in sector s as a function of the bubble in sector $s' \neq s$, i.e. $x_D^*(x_{Ht})$ and $x_H^*(x_{Dt})$. If, as we have assumed, the dot-com bubble is expansionary while the housing bubble is contractionary, it can be verified that x_D^* is increasing in x_{Ht} while x_H^* is increasing in x_{Dt}. The intuition behind this result is quite straightforward. The total demand for bubbles comes from dynamically inefficient investments. Expansionary bubbles raise the capital stock, lowering the interest rate and extending dynamic inefficiency. Conventional or contractionary bubbles instead lower the capital stock, raising the interest rate and eliminating dynamically inefficient investments. These two types of bubbles thus offset each other's effects and, in doing so, they complement and reinforce one another.

Figure 7 plots the dynamics of x_{Dt} and x_{Ht}. The solid upward-sloping loci depict the stationary bubbles in both sectors, $x_D^*(x_{Ht})$ and $x_H^*(x_{Dt})$, and their intersection represents the bubbly steady-state of the model.[20] The figure represents a dynamically efficient economy in which contractionary bubbles cannot exist on their own: this is why $x_H^*(0) < 0$. It also depicts combinations of x_{Dt} and x_{Ht} that satisfy Equation (24).

Figure 7 can be used to analyze the interaction between these bubbly episodes. Consider that there is initially a growing expansionary bubble in the dot-com sector, so that $x_{Dt} < x_D^*(0)$. When bubble x_{Ht} appears in the housing sector, it contracts the output of both sectors. But it also boosts the growth of x_{Dt}, directly enhancing the productivity of the dot-com sector: hence, there is positive contagion as the bubble "spreads" across sectors and exacerbates the overvaluation of firms throughout the economy. The stable path in the figure illustrates the evolution of both bubbles in this case. Of course, the opposite effects are at play when a bubbly episode ends. Suppose that investors become pessimistic regarding the evolution of the dot-com sector and the expansionary bubble in the sector collapses. Then, output falls in both sectors. But the collapse also generates contagion effects and it spreads to the housing bubble, which must also contract. And if, as depicted in the figure, the economy is dynamically efficient in the fundamental state, the housing bubble can no longer exist and it necessarily collapses as well.[21] Like a stack of dominoes, the fall of one bubble takes the other one with it.

It is also possible for bubbly episodes to substitute one another generating "negative contagion". This happens when these episodes are of the same type. To see this, assume that there is an expansionary bubbly episode in the dot-com sector and an expansionary one in the housing sector for which

$$\frac{\phi \cdot (\pi - 1) \cdot n_H}{1 - \phi \cdot \pi} > 1. \tag{27}$$

We can once more find $x_D^*(x_{Ht})$ and $x_H^*(x_{Dt})$ using Equation (23) and verify that, if both episodes are expansionary, x_D^* is decreasing in x_{Ht} while x_H^* is decreasing in x_{Dt}. Intuitively, expansionary bubbles raise the capital stock and lower the interest rate, thereby decreasing the rate at which other bubbles must grow in order to be attractive. Hence, when an expansionary bubble appears in sector $s \in \{D, H\}$, it decreases the growth rate – and thus the equilibrium size – of expansionary bubbles in sector $s' \neq s$.

Figure 8 below plots the dynamics of x_D and x_H in this case. The solid loci depict the stationary bubbles in both sectors, $x_D^*(x_{Ht})$ and $x_H^*(x_{Dt})$, so that their intersection represents the bubbly

[20] Figure 6 depicts the particular case in which $\dfrac{1+n_D}{1-p_D(x_D^*(0))} > \dfrac{1+n_H}{1-p_H(0)}$.

[21] This is, once again, because conventional or contractionary bubbles can only arise in the presence of dynamic inefficiency.

steady-state of the model. Once again, the figure depicts only combinations of x_{Dt} and x_{Ht} that satisfy Equation (24) and it assumes that there is a bubbly steady state within this range.

In this case, the dynamics of the economy are globally stable: regardless of the initial values of x_{Dt} and x_{Ht}, the economy converges to the steady state. As before, consider that there is initially a growing expansionary bubble in the dot-com sector, so that $x_{Dt} < x_D^*(0)$. When bubble x_{Ht} appears in the housing sector, it expands the output of both sectors. But it also dampens the growth rate of the dot-com bubble and decreases its stationary size relative to $x_D^*(0)$. In this sense, there is negative contagion since the rise of the housing bubble partially crowds out the dot-com bubble. Clearly, the opposite effects follow the bursting of any of these bubbles. If investors become pessimistic regarding the prospects of the dot-com sector, its bubble – and thus, the value of its firms – collapses. But this collapse now feeds the housing bubble, which increases its growth rate and its share in the portfolios of non-entrepreneurs. Once again, there is negative contagion as the bubble shifts from one sector to the other.

Finally, we would like to note that the results in this section point towards interesting applications in international economics. We have referred throughout to an economy with multiple sectors. But, with minor modifications, this same model can be used to analyze a world economy composed of multiple countries. In the simplest of such worlds, each sector could be interpreted as a different country. This interpretation would be perfectly consistent with our analysis here under the assumptions of: (i) international financial integration, which guarantees that interest rates are equalized across countries, and; (ii) international trade, which can be used to guarantee that wage rates are also equalized across countries.[22] In this case, the examples analyzed above could be used to think about the speed and intensity with which bubbly episodes seem to spread across countries.[23]

[22] A simple way of doing this is to assume the existence of a second layer of intermediate goods, that we call K- and L-goods. A K-good is an intermediate good that is produced with one unit of capital, whereas an L-good is produced with one unit of labor. In this case, trade in intermediates leads to factor-price equalization and all of our results apply immediately.

[23] Or to study the connection between international capital flows and bubbles. Ventura (2004) argues that restrictions to capital flows fuel bubbles. Caballero, Farhi and Gourinchas (2008) argue that, due to their relatively low level of financial development (i.e. low ϕ), fast-growing Asian economies supply savings but little intermediation to the world economy. By doing so, they boost demand for assets in developed economies and relax the conditions for the existence of asset bubbles.

4 Policy implications

We have modeled the crisis as a negative shock to net worth that led to a collapse of intermediation and the average efficiency of investment. Is there anything that governments can do to reverse such a situation? If the shock is fundamental or technological, the canonical model cannot provide a meaningful answer to this question since it lacks a good description of the microeconomics of productivity and the financial friction. But if the shock is the bursting of a bubble, the canonical model turns out to be quite useful for policy analysis. Keeping with the exploratory spirit of these notes, we add a government to the framework developed above and draw some tentative results.

4.1 Setup with a government

Assume next that the world economy contains a government that gives subsidies to firms and finances these subsidies by taxing individuals and issuing debt. Unlike much of the recent literature on the crisis, we do not to give the government an advantage over the market as a lender. Instead, we assume the government enforces payments due using the same legal system and related institutional arrangements as the private sector.[24] This implies that it is not possible to improve the allocation of investments without raising the net worth of new firms.[25] To save on notation, we return to the one-sector model of sections 1 and 2.

Let T_{it} and S_{jt} be the tax levied on individual i and the subsidy given to firm j in period t. The government borrows by issuing one-period bonds which yield a (gross) return equal to R^d_{t+1}. As in the case of private debt, we allow this return to vary across states of nature. This could reflect a contingent contractual rate, or the government's failure to keep with its contractual obligations. Let d_t be the payments made to debtholders in period t. Then, the government's budget constraint can be written as follows:

$$d_{t+1} = R^d_{t+1} \cdot (d_t + S_t - T_t)\,, \tag{28}$$

where $T_t \equiv \int_{i \in I_t} T_{it}$ and $S_t \equiv \int_{j \in J_t} S_{jt}$. Equation (28) says that the government borrows to make debt payments, i.e. d_t, and to finance the budget deficit, i.e. $S_t - T_t$.

[24] For instance, some of the policies advocated by Gertler and Kiyotaki (2010) and Curdia and Woodford (2010) are based on the assumption that (at least, after the crisis) the government is better at lending than the private sector.

[25] Consider a proposal for the government to lend to new firms. Since the total amount of resources that the legal system can extract from these firms is fixed, any lending done by the the government uses up an equivalent amount of net worth. If financed by issuing debt and/or taxing non-entrepreneurs, government lending crowds out private credit one-to-one. Even worse, if partly financed by taxing entrepreneurs, government lending crowds out private credit more than one-to-one. The reason is that taking away resources from entrepreneurs lowers the net worth of their firms.

The presence of the government has no effect on technology, i.e. Equations (1) and (2); or the functioning of the labor market, i.e. Equations (3) and (4). It does however affect the financial market in three specific ways: (i) there is now an additional market for government debt; (ii) taxes reduce the savings available to purchase financial assets; and (iii) subsidies improve the balance sheet of firms and therefore their net worth. This last effect means that Equation (5) should be replaced by the following one:

$$R_{t+1} \cdot f_{jt} \leq \phi_{t+1} \cdot \left[F\left(l_{jt+1}, k_{jt+1}\right) - w_{t+1} \cdot l_{jt+1} + S_{jt} + V_{jt+1}\right]. \tag{29}$$

Equation (29) recognizes that future subsidies also constitute a source of revenue for the firm. The conditions for maximization also need to be modified as follows:

$$E_t R_{t+1} = E_t R^d_{t+1} = \max_{\langle I_{jt}, f_{jt}\rangle} \frac{E_t\left\{\alpha \cdot k_{t+1}^{\alpha-1} \cdot \left[A_{jt} \cdot I_{jt} + (1-\delta) \cdot k_{jt}\right] - R_{t+1} \cdot f_{jt} + S_{jt} + V_{jt+1}\right\}}{V_{jt} + I_{jt} - f_{jt}} \quad \text{if } j \notin J_t^N, \tag{30}$$

$$E_t R_{t+1} \leq \max_{\langle I_{jt}, f_{jt}\rangle} \frac{E_t\left\{\alpha \cdot k_{t+1}^{\alpha-1} \cdot A_{jt} \cdot I_{jt} - R_{t+1} \cdot f_{jt} + S_{jt} + V_{jt+1}\right\}}{I_{jt} - f_{jt}} \quad \text{if } j \in J_t^N. \tag{31}$$

Equations (30) and (31) are natural generalizations of Equations (6) and (7). Equation (30) says that maximization by entrepreneurs requires that the expected return to owning an old firm and holding government debt must equal the interest rate. Equation (31) says that maximization by entrepreneurs implies that starting new firms must yield a return that is at least as high as the interest rate.

We conjecture that firm prices and the interest rate on private credit are still given by Equations (13) and (8), respectively. In addition, we conjecture that the expected return on government debt is given by:

$$E_t R^d_{t+1} = \alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta. \tag{32}$$

Equation (32) says that government debt must offer the same expected return as private credit. This is a direct implication of risk neutrality.

At the proposed interest rate and firm prices, entrepreneurs strictly prefer to start new firms than to lend or purchase old firms and, just as before, they ask for as much credit as possible:

$$f_{jt} = \frac{1}{1 - E_t \phi_{t+1} \cdot \pi_t} \cdot E_t\left\{\phi_{t+1} \cdot \left(\pi_t \cdot (w_t - T_{it}) + \frac{b_{jt+1} + S_{jt+1}}{\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta}\right)\right\}, \tag{33}$$

where T_{it} are the taxes levied on the entrepreneur that starts and owns firm j. Intermediation decreases with taxes on entrepreneurs and increases with subsidies to new firms.

At the proposed interest rate and firm prices, non-entrepreneurs are indifferent among lending to new firms, buying government debt or purchasing old firms. If they choose the latter, they are also indifferent regarding the amount of investment and external financing of their firms. As a group, the non-entrepreneurs purchase the stock of old firms, give credit to new firms, buy the government debt and use any savings left to produce new capital within their old firms. To verify that markets clear, we must check now that:

$$(1-\varepsilon)\cdot w_t - \left(T_t - T_t^E\right) - f_t^N - (d_t + S_t - T_t) \geq V_t, \tag{34}$$

where $T_t^E \equiv \int_{i \in I_t^E} T_{it}$. We keep assuming that this condition holds and our conjecture is verified.[26]

Aggregating Equation (1) across firms, we find that:[27]

$$k_{t+1} = \left[1 + \frac{(\pi_t - 1)\cdot\varepsilon}{1 - E_t\phi_{t+1}\cdot\pi_t}\right]\cdot(1-\alpha)\cdot k_t^\alpha + \frac{\pi_t - 1}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot\left(\frac{E_t\left\{\phi_{t+1}\cdot\left(b_{t+1}^N + S_{t+1}^N\right)\right\}}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta} - T_t^E\right) - b_t - b_t^N - d_t - S_t, \tag{35}$$

where $S_t^N \equiv \int_{j \in J_{t-1}^N} S_{jt}$. A comparison of Equations (16) and (35) shows that fiscal policy has two effects on capital accumulation. The first one is the conventional crowding-out effect, captured by the last two terms of Equation (35). As the debt grows, it absorbs a larger fraction of the savings of the young generation and this diverts resources away from capital accumulation. But there is also a second effect here that is due to the financial friction and is captured by the second term of Equation (16). Subsidies to new firms foster capital accumulation by relaxing credit constraints, increasing intermediation and the average efficiency of investment. For the opposite reasons, taxes to entrepreneurs reduce capital accumulation.

To complete the description of the dynamics of the economy, we still need Equation (17) describing the evolution of the aggregate bubble and, in addition, we need the following equation

[26] This requires now that:

$$\frac{1 - E_t\phi_{t+1}\cdot\pi_t - \varepsilon}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot(1-\alpha)\cdot k_t^\alpha + \frac{1}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot\left(\frac{E_t\left\{\phi_{t+1}\cdot\left(b_{t+1}^N + S_{t+1}^N\right)\right\}}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta} - T_t^E\right) - (d_t + S_t) \geq (1-\delta)\cdot k_t + b_t + b_t^N,$$

where $T_t^N \equiv \int_{j \in J_{t-1}^N} T_{jt}$. Note that taxes on entrepreneurs relax this condition while debt and subsidies tighten it.

[27] Investment spending consists of the savings of the young minus their purchases of old firms and government debt, i.e. $w_t - T_t - V_t - (d_t + S_t - T_t) = (1-\alpha)\cdot k_t^\alpha - (1-\delta)\cdot k_t - b_t - b_t^N - d_t - S_t$. Of this total, new firms invest $\frac{1}{1 - E_t\phi_{t+1}\cdot\pi_t}\cdot\left(\varepsilon\cdot(1-\alpha)\cdot k_t^\alpha + \frac{E_t\left\{\phi_{t+1}\cdot\left(b_{t+1}^N + T_{t+1}^N\right)\right\}}{\alpha\cdot k_{t+1}^{\alpha-1} + 1 - \delta} - T_t^E\right)$ with efficiency π_t, while the rest is invested by old firms with efficiency one.

describing the evolution of fiscal variables:

$$E_t d_{t+1} = \left(\alpha \cdot k_{t+1}^{\alpha-1} + 1 - \delta\right) \cdot (d_t + S_t - T_t)\,. \tag{36}$$

Equation (36) follows from Equations (28) and (32). The equilibrium depends on the fiscal policy adopted by the government. A fiscal policy is a feasible sequence for taxes and subsidies, i.e. T_{it} and S_{jt}, and a return process R_{t+1}^d satisfying Equation (32). Once this policy has been specified, any sequence for k_t, d_t, b_t and b_t^N that satisfies Equations (17), (35) and (36) is an equilibrium, provided that Equation (34) holds in all dates and states of nature. We show next how fiscal policy works in some of these equilibria.

4.2 'Undoing' the crisis?

Let us start with a disclaimer: we do not search for the optimal fiscal policy. Instead, we focus on the more modest question of whether the government can use fiscal policy to reverse the situation and bring the economy back to the pre-crisis path. This might be a desirable goal for most individuals, but not necessarily for all as some might benefit from the crisis. Moreover, the pre-crisis path might not be the optimal path in any meaningful way. To determine the optimal path, we need to give weights to the welfare of different individuals by choosing a social welfare function. We do not do this here.

The key observation is that the bubble implements a series of intragenerational and intergenerational transfers that the government can replicate with fiscal policy. In fact, Equations (17), (35) and (36) provide a simple blueprint for fiscal policy to undo the crisis:

1. Set all fiscal variables equal to zero, i.e. $T_{it} = S_{jt} = d_t = 0$ for all $i \in I_t$ and $j \in J_t$, and use Equations (17) and (35) to describe the desired bubbly equilibrium. Let $\hat{b}_t^N$ and $\hat{b}_t$ describe this equilibrium.

2. Then, set the following targets for fiscal variables: (i) $T_{it} = 0$ for all $i \in I_t^E$; (ii) $S_t^N = \hat{b}_t^N - b_t^N$; and (iii) $d_t = \hat{b}_t - b_t$.

This simple algorithm describes the fiscal policy that replicates the desired bubbly equilibrium. Since $\hat{b}_t^N$ and $\hat{b}_t$ are an equilibrium of the economy without fiscal policy, the proposed fiscal policy is always feasible. Note however that this policy is not fully determined, since target (iii) can be achieved through various combinations of taxes to non-entrepreneurs, $T_t - T_t^E$; subsidies to old firms, $S_t - S_t^N$; and returns to public debt R_{t+1}^d. These alternatives have different distributional implications but the same implications for the path of debt and the capital stock.

We have now a blueprint to restore the pre-crisis path. Assume, for instance, that the economy was close to the bubbly steady state in Figure 5 before the bursting of the bubble. In this steady state, we have that:

$$b_t = \frac{\frac{\alpha}{1-\alpha} \cdot \frac{1+n}{1-p} - \left(1 + \frac{(\pi-1)\cdot\varepsilon}{1-\phi\cdot\pi}\right)}{\left(\frac{(\pi-1)\cdot\phi\cdot n}{1-\phi\cdot\pi} - 1\right)\cdot(1+n)} \cdot (1-\alpha)\cdot k_t^{\alpha},$$

and, of course, $b_t^N = n \cdot b_t$. When the bubble bursts, the government can restore the pre-crisis path by issuing an amount of debt that replaces the bubble, and giving subsidies to new firms that amount to a fraction n of this debt. If $n < 1$, the additional proceeds from borrowing can be used to subsidize old firms.[28] If $n > 1$, the government would have to raise taxes on non-entrepreneurs to finance the subsidies.

Going beyond this simple example, the government can restore any desired bubbly equilibrium by replacing the bubble with government debt and rolling it over until the bubble pops up again. Then, the government lowers government debt by reducing subsidies and raising taxes. This simple counter-cyclical fiscal policy stabilizes the economy when there are shocks to investor sentiment.

This result seems quite strong and should raise some suspicion. Can the government really use fiscal policy to undo changes in investor sentiment? Or have we cheated somewhere along the argument? The answer to both questions is partly affirmative. A crucial implicit assumption is that the government can commit to this type of fiscal policy. If this is the case, then the government can undo changes in investor sentiment following the blueprint above. If the government cannot commit, the question becomes much more complicated.

Assume the markets test the government, and refuse to buy the debt based on the belief that the government will not pay it back. This makes it impossible for the proposed fiscal policy to continue, lowering the net worth of new firms, reducing intermediation and the average efficiency of investment. In a nutshell, this brings the crisis back. In such a scenario, will the government pay back the debt? Or will the government instead default and validate the belief of the market? Paying back the debt requires forfeiting on promised subsidies to new firms and taxing the young

[28] More realistically, they could be used to lower taxes. Here this is not possible because we have set taxes to zero before the crisis. Adding government spending to the model would take us away from this artificial corner.

to pay the debtholders. These measures would further reduce capital accumulation and make the crisis even worse. Although we have not modelled government objectives, it seems reasonable to think that the temptation to default would be high.

So where are we now? Can the government really undo the crisis with fiscal policy? If the government has commitment, the answer is unambiguously affirmative. If the government has no commitment, it might still try. Whether it succeeds or not depends again on market expectations. If the market is pessimistic, fiscal policy might be ineffective or, even worse, it might deepen the crisis. In this case, fiscal policy transforms a severe financial crisis into an even worse sovereign debt crisis.

5 Concluding remarks

These notes have explored a view of the current crisis as a shock to investor sentiment that led to the collapse of a bubble or pyramid scheme in financial markets. According to this view, asset prices today depend on market expectations of future asset prices. When investor sentiment is high, asset prices are high and this raises the net worth of firms, relaxing their credit constraints and improving the allocation of investment. This fosters capital accumulation and consumption. When investor sentiment is low, the opposite occurs: lower asset prices reduce the net worth of firms, tightening their credit constraints and worsening the allocation of investment. This leads to a reduction in credit, output and consumption.

As a research strategy, viewing the current crisis as the collapse of a bubble is more appealing than alternatives that rely on fundamental or technological shocks. It provides a simple unified narrative of the main macroeconomic developments of the recent past and the current crisis. Namely, the crisis was caused by the collapse of a bubbly episode that had sustained a steady expansion in net worth, output and consumption since the 1990s. This narrative is consistent with the fact that the expansionary phase was gradual and protracted while the recessionary phase has been sudden and sharp. It does nor require us to identify a large and negative fundamental or technological shock to blame for the current state of the world economy. It can also account for the connection (or lack of connection!) between financial and real economic activity, and the speed and strength with which the crisis has been transmitted across different sectors or countries. Finally, it provides us with a simple blueprint for the design of fiscal policies that 'undo' the crisis, although it also highlights that these policies rely on government commitment for their success.

The analytical framework developed in these notes allows us to think through various aspects of the current crisis. But there are a couple of very important loose ends. The crisis has led to a significant increase in unemployment throughout the world. Our model, with flexible wages and a fully inelastic labor supply, has nothing to say about this. The crisis has also had a strong international dimension to it, as it took place in a context of global imbalances and it has led to the steepest fall of world trade in recorded history. Our model of the world economy is too rough and simple to speak to these issues in a meaningful way. A satisfactory treatment of unemployment and international trade requires a fully-fledged multi-country model with realistic frictions in labor markets. Building such a model should be the next step in this research program.

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6 Appendix

This section derives the wage and the sectorial productions as a function of the aggregate stock of capital. Let $p_t = \dfrac{p_{Dt}}{p_{Ht}}$ denote the relative price of dot-com to housing at time t. On the consumption side, the Cobb-Douglas preferences imply that in equilibrium: (i) expenditure in both goods is equalized at all points in time, so that $p_t = \dfrac{y_{Ht}}{y_{Dt}}$, and; (ii) given that the price of the composite good is normalized to one, $\eta \cdot (2 \cdot p_{Ht})^{\frac{1}{2}} \cdot (2 \cdot p_{Dt})^{\frac{1}{2}} = 1$.

On the production side, cost minimization by firms in either sector yields:

$$\left(\frac{w_t}{1-\alpha_H}\right)^{1-\alpha_H} \cdot \left(\frac{\rho_t}{\alpha_H}\right)^{\alpha_H} = \frac{1}{2} \cdot p_t^{-\frac{1}{2}} = p_{Ht}, \tag{37}$$

$$\left(\frac{w_t}{1-\alpha_D}\right)^{1-\alpha_D} \cdot \left(\frac{\rho_t}{\alpha_D}\right)^{\alpha_D} = \frac{1}{2} \cdot p_t^{\frac{1}{2}} = p_{Dt}, \tag{38}$$

where ρ_t is the rental price of capital and we have assumed that factor prices are the same across sectors, i.e. that there is perfect mobility of labor across sectors and that there are non-entrepreneurs investing in both sectors. We can apply Shepard's lemma and derive Equations (37) and (38) to obtain factor demands. This delivers the following market-clearing conditions for capital and labor:

$$1 = (1-\alpha_H) \cdot \frac{\frac{1}{2} \cdot p_t^{-\frac{1}{2}}}{w_t} \cdot y_{Ht} + (1-\alpha_D) \cdot \frac{\frac{1}{2} \cdot p_t^{\frac{1}{2}}}{w_t} \cdot y_{Dt}, \tag{39}$$

$$k_t = \alpha_H \cdot \frac{\frac{1}{2} \cdot p_t^{-\frac{1}{2}}}{\rho_t} \cdot y_{Ht} + \alpha_D \cdot \frac{\frac{1}{2} \cdot p_t^{\frac{1}{2}}}{\rho_t} \cdot y_{Dt}, \tag{40}$$

where k_t denotes the economy's aggregate stock of capital. Using both equations jointly with the consumption-side implications for p_t in (i) and (ii) above delivers the following expression for the production of the composite good:

$$y_t = \eta \cdot (y_{Ht})^{\frac{1}{2}} \cdot (y_{Dt})^{\frac{1}{2}} = k_t^{\alpha}, \tag{41}$$

where $\alpha = \dfrac{\alpha_H + \alpha_D}{2}$ is an average of the capital shares in both sectors, and we have chosen units to make

$$\eta = 2 \cdot \left[\left(\frac{1-\alpha}{1-\alpha_H}\right)^{\frac{1-\alpha_H}{2}} \cdot \left(\frac{\alpha}{\alpha_H}\right)^{\frac{\alpha_H}{2}} \cdot \left(\frac{1-\alpha}{1-\alpha_D}\right)^{\frac{1-\alpha_D}{2}} \cdot \left(\frac{\alpha}{\alpha_D}\right)^{\frac{\alpha_D}{2}} \right].$$

Equation (41) thus shows how, despite the multi-sector structure of this model, output of the composite good is a simple function of the economy's aggregate capital stock. Note that, since

Equation (41) has been derived under the assumption that the marginal product of labor is equalized across sectors, it follows that the wage is still given by Equation (4).

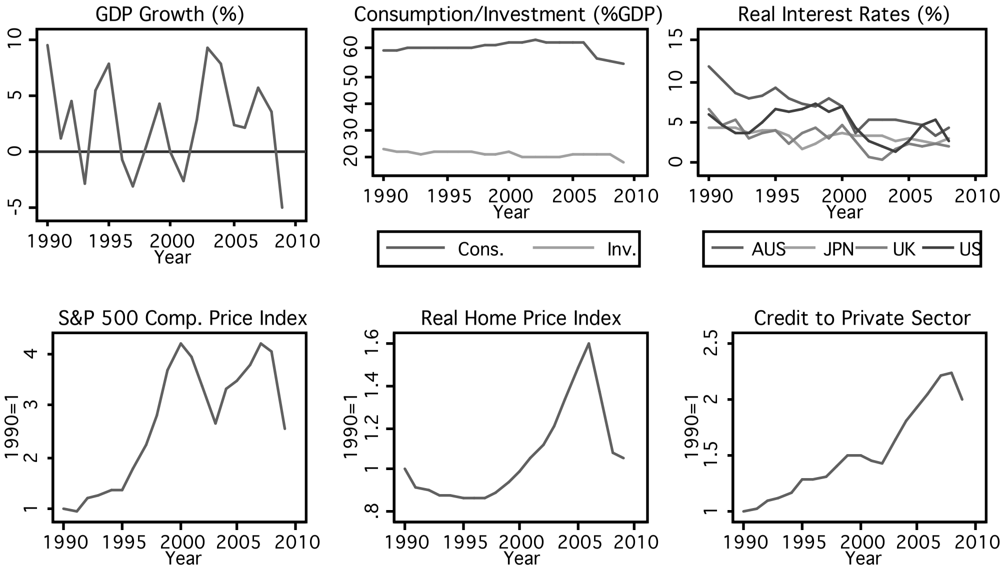
To retrieve the sectorial productions, we first use Equations (37)-(40) to express p_t in terms of k_t. Replacing this expression in Equation (41) yields Equation (20):

$$y_{st} = A_s \cdot k_t^{\alpha_s}, \text{ for } s \in \{D, H\},$$

where A_s is a sector-specific constant

$$A_s = \frac{1}{2} \cdot \left(\frac{1-\alpha_s}{1-\alpha}\right)^{1-\alpha_s} \cdot \left(\frac{\alpha_s}{\alpha}\right)^{\alpha_s} \quad \text{for} \quad s \in \{D, H\}.$$

Figure 1: Recent Macroeconomic Trends in Advanced Economies



Notes:
Real variables constitute aggregates of data from Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Slovak Republic, Spain, Sweden, Switzerland, United Kingdom, United States. Data is taken from the IMF's WEO and the World Bank's WDI. Financial indeces (for the United States only) are based on Shiller (2005).

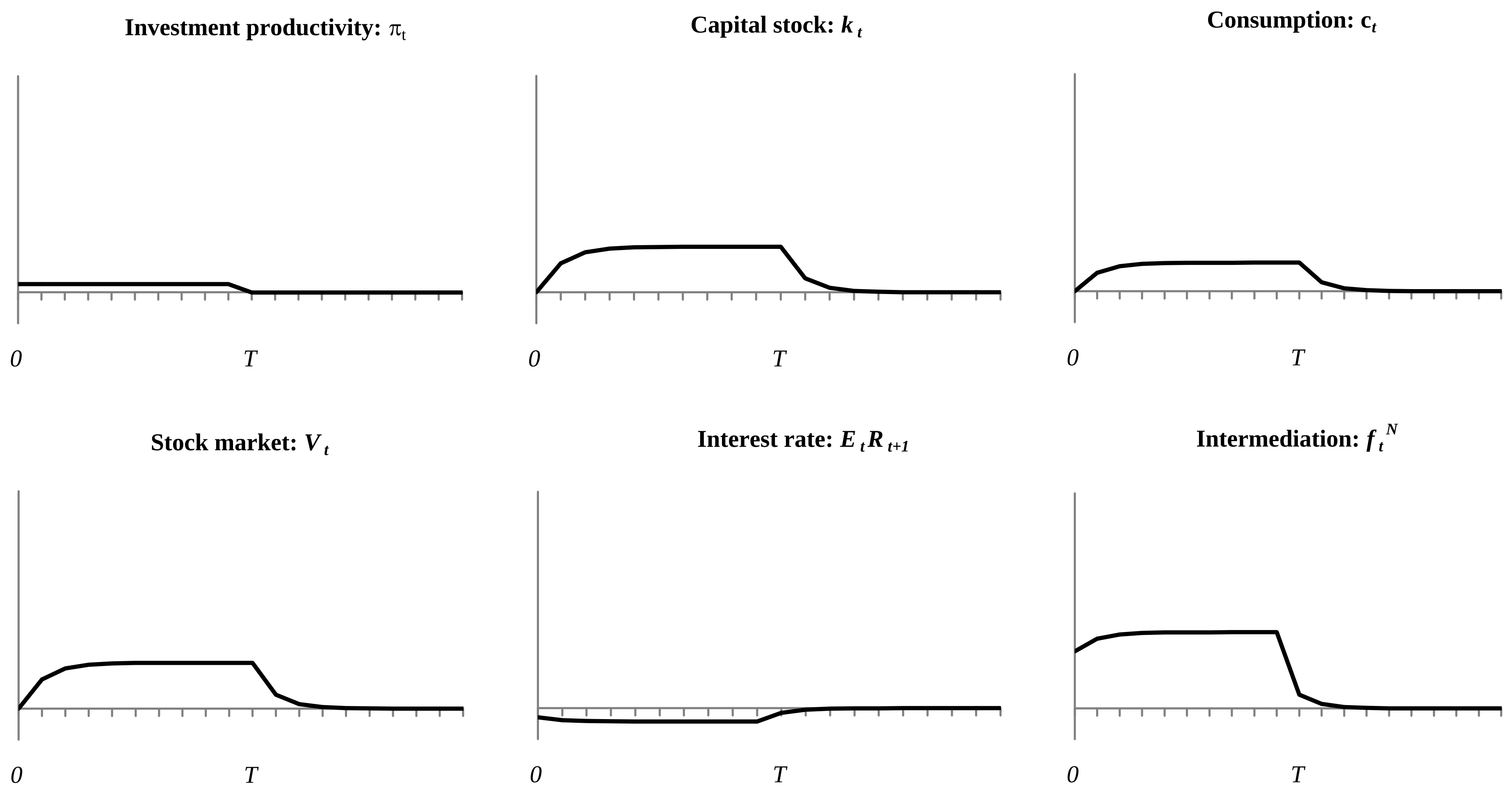


Figure 2: Transitory Shock to Investment Productivity in the Canonical Model

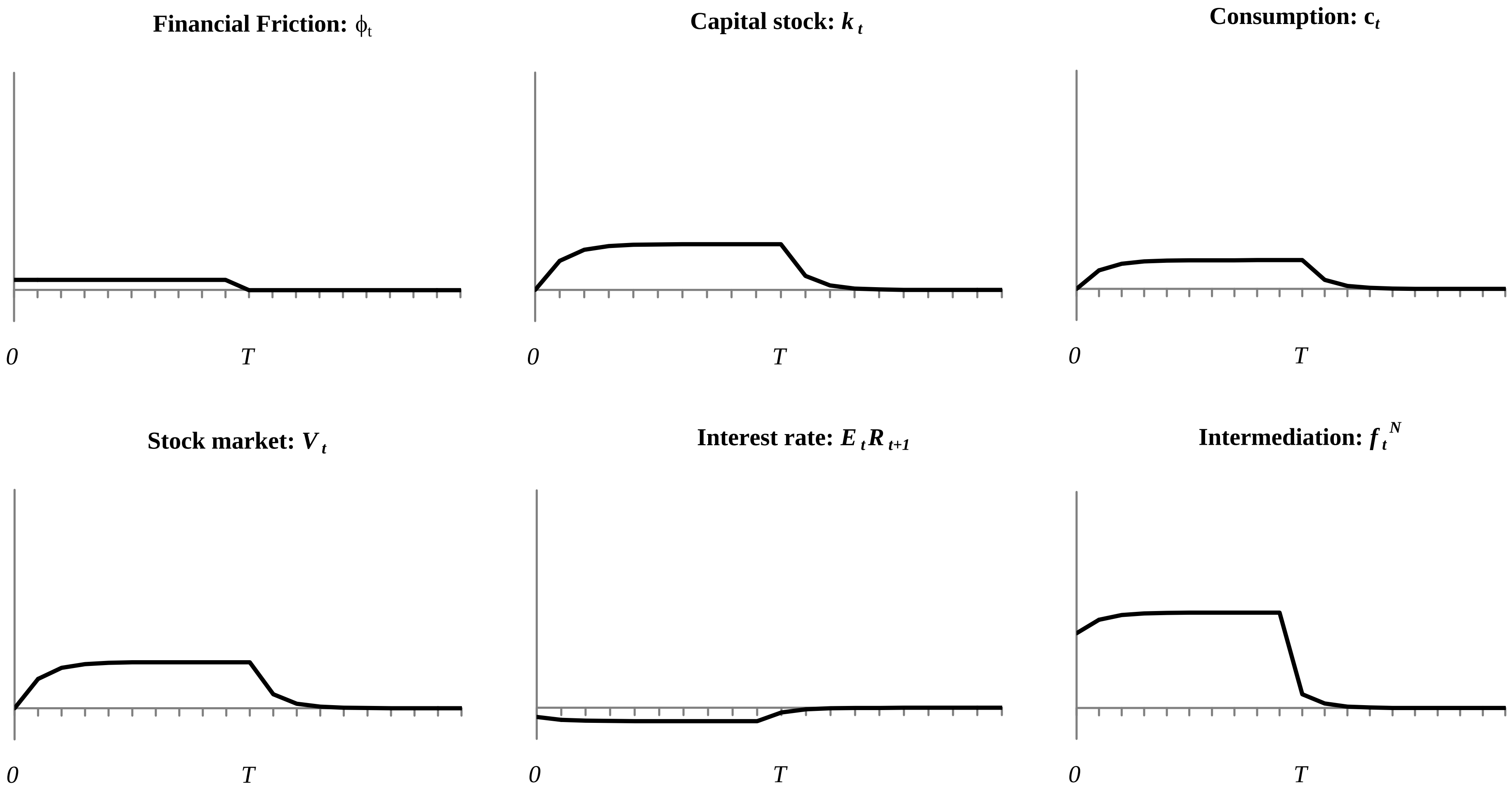


Figure 3: Transitory Shock to the Financial Friction in the Canonical Model

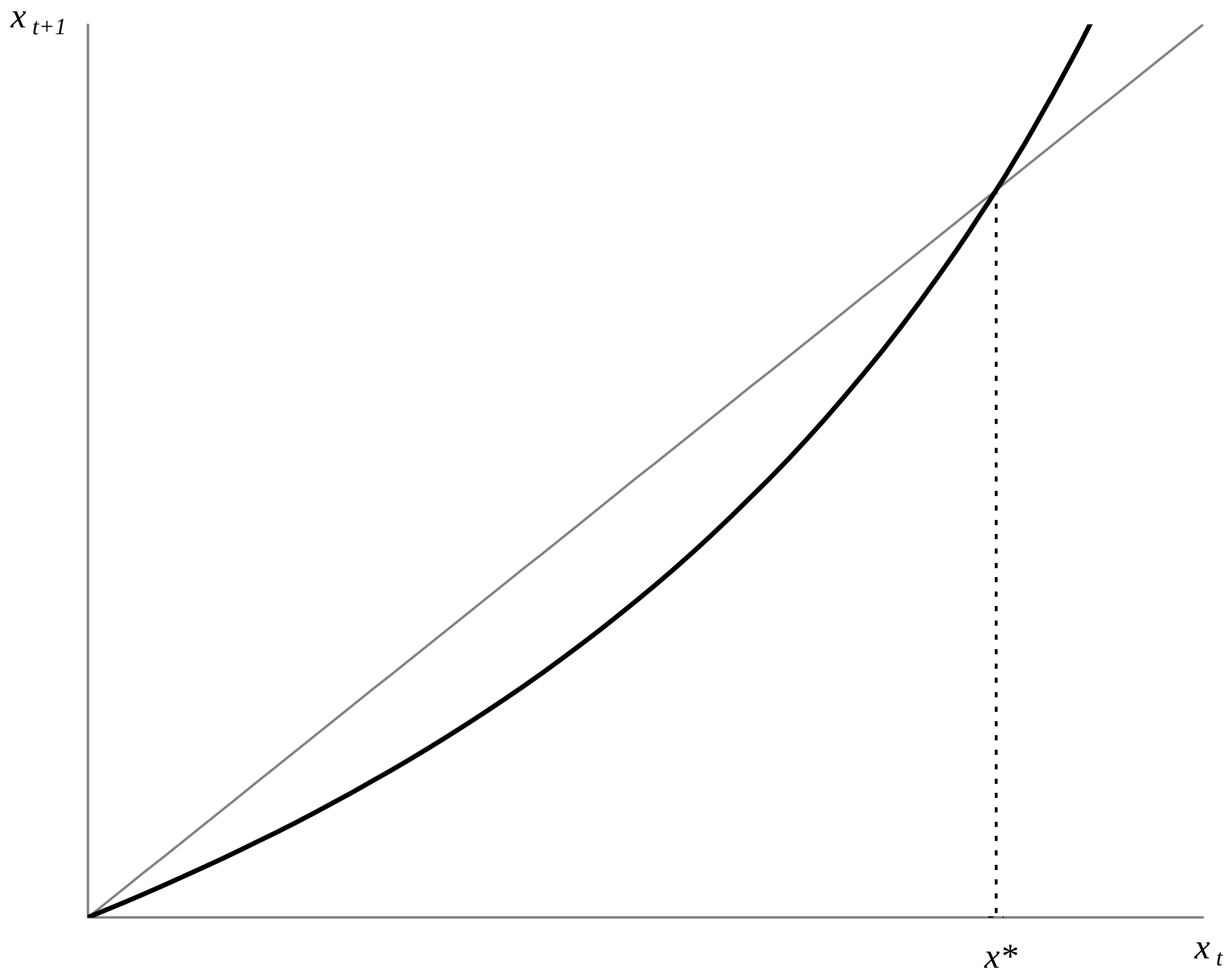


Figure 4: Contractionary Bubble

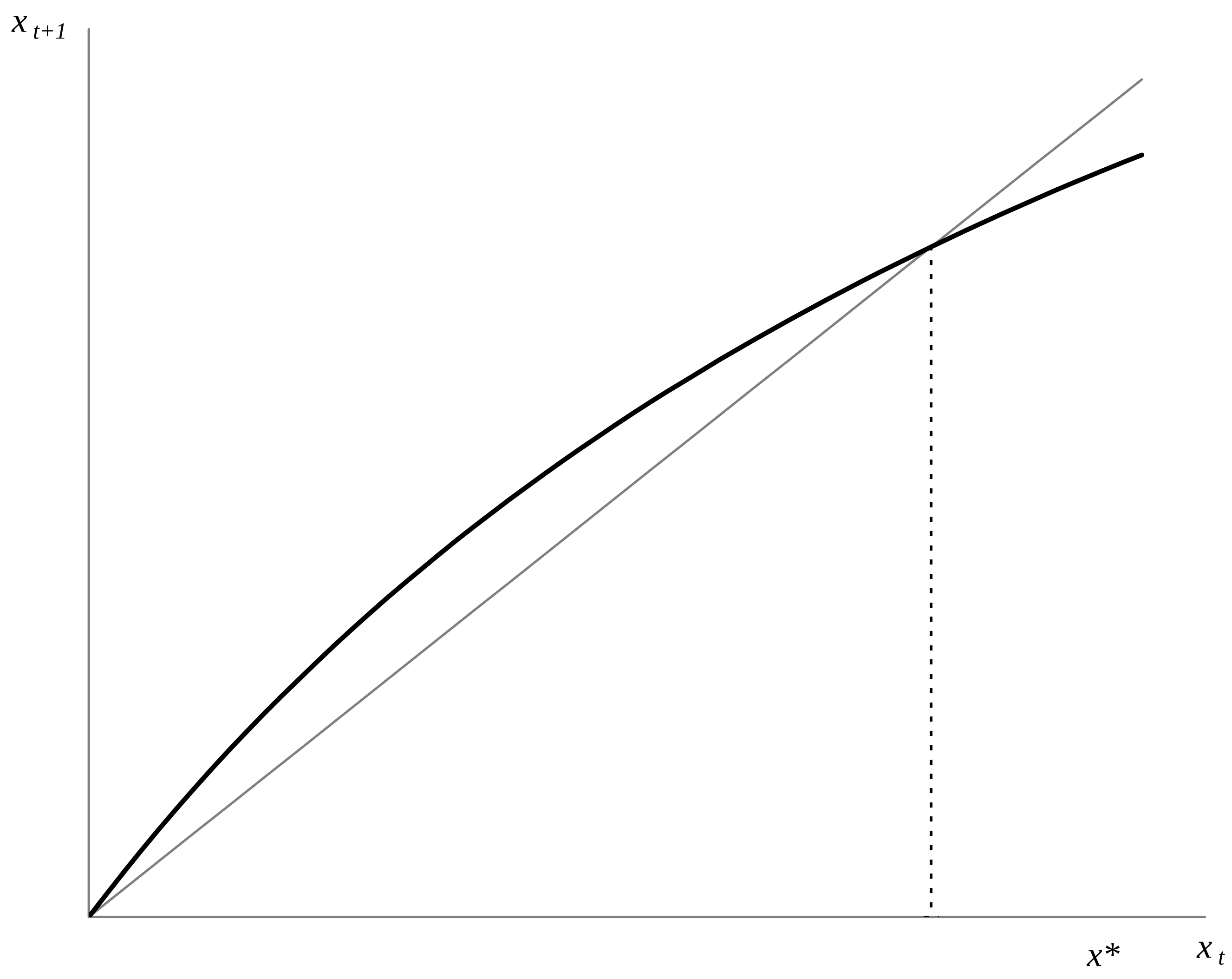


Figure 5: Expansionary Bubble

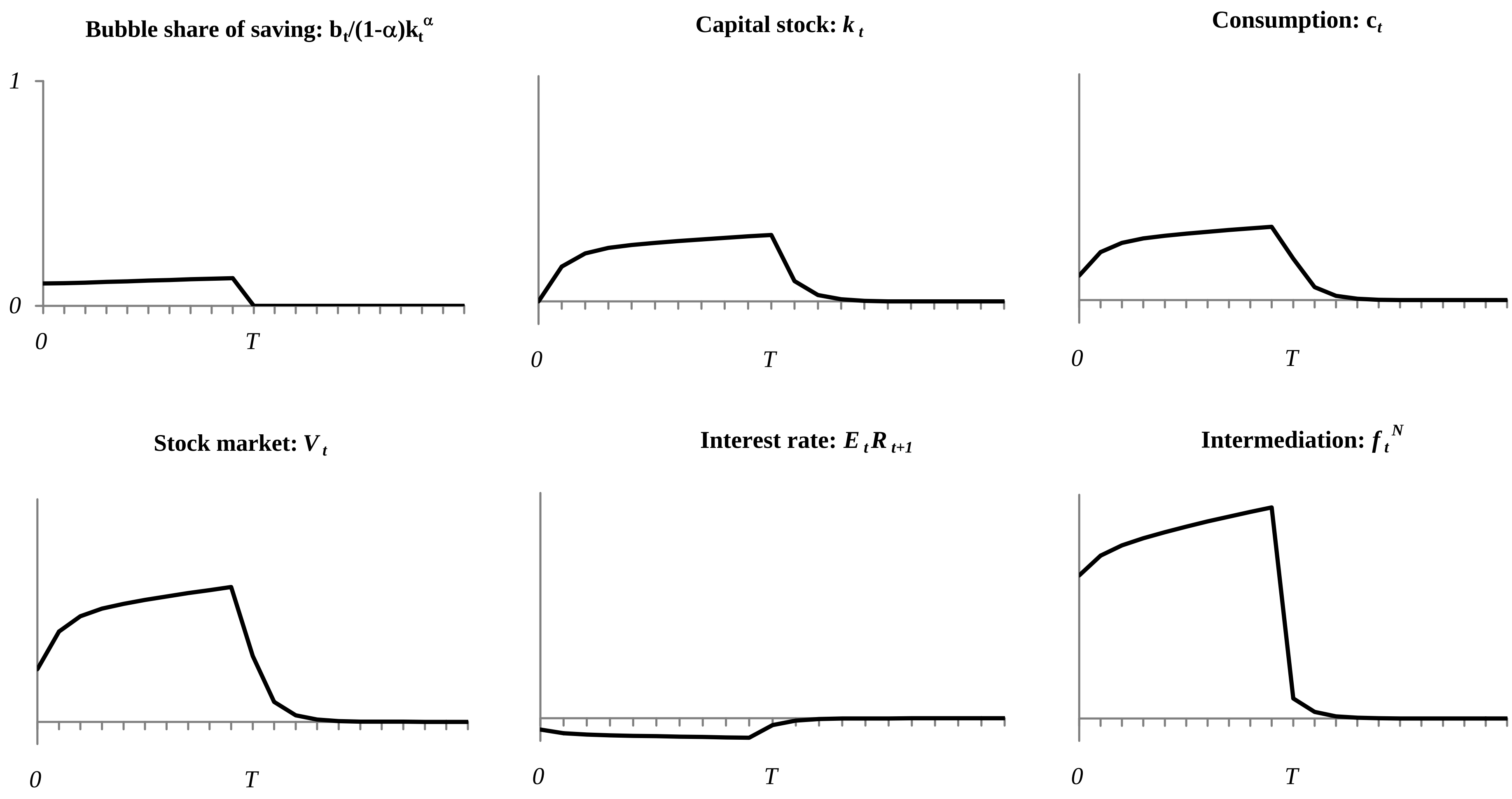


Figure 6: Transitory Expansionary Bubble

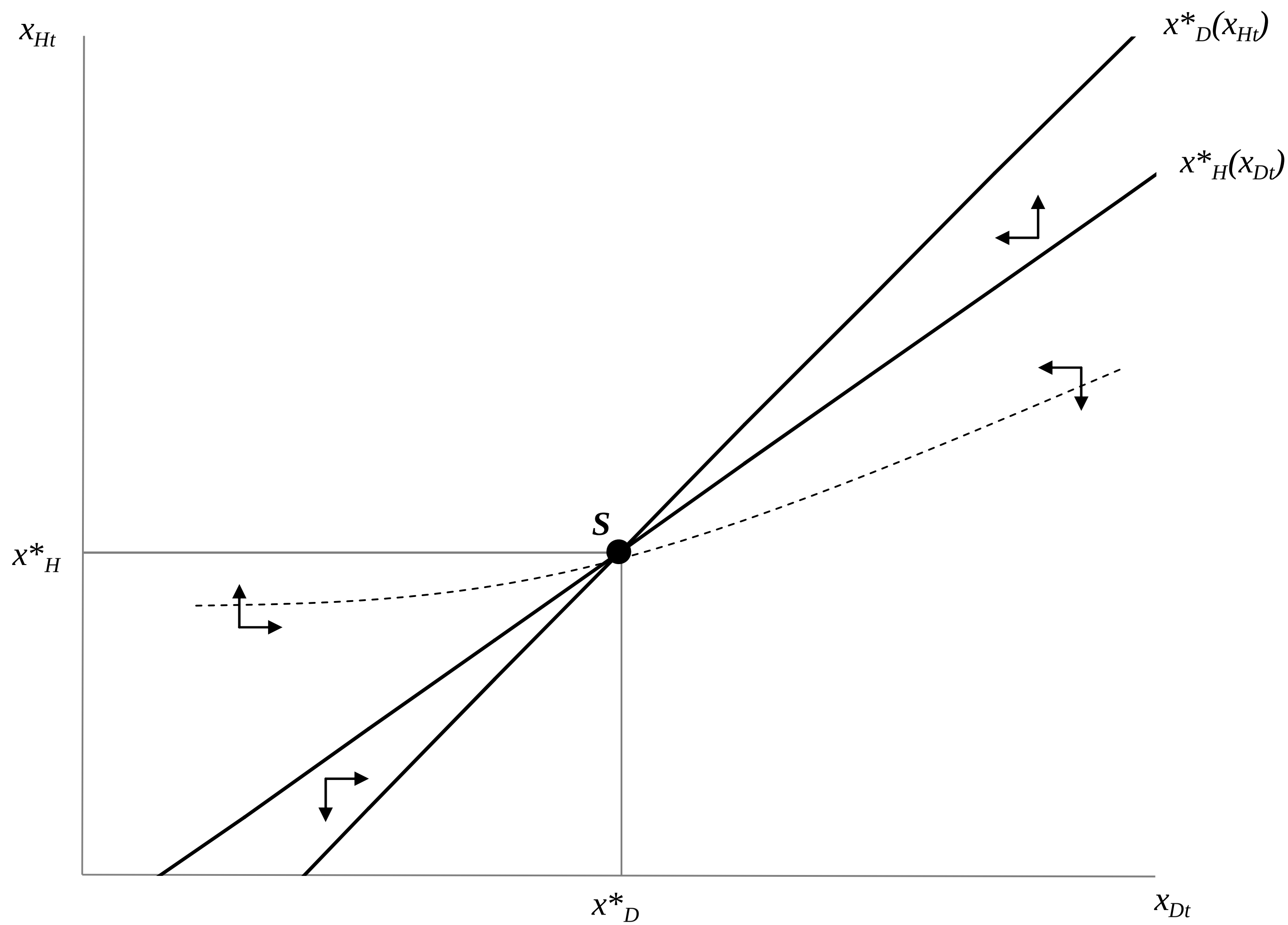


Figure 7: Expansionary and Contractionary Bubble

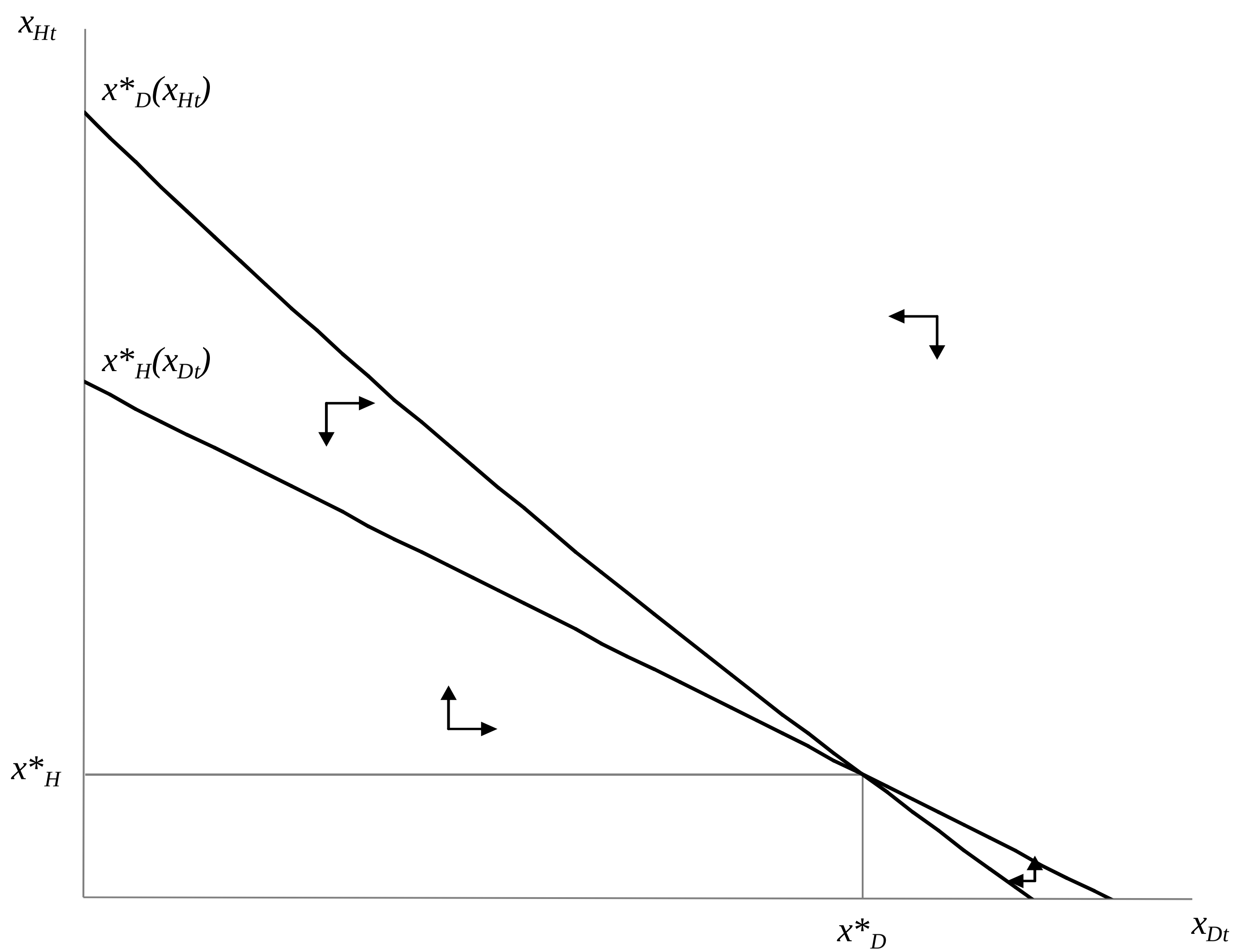


Figure 8: Two Expansionary Bubbles